

Sec (12) ماتري

Sheet (10) → Three phase system

17 - الماتري للثلاث مرات

# Threephase

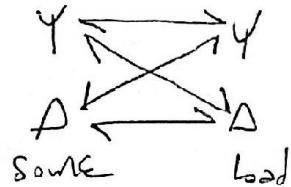
1

17 ← 18 circuit

3-Sources Connected to 3 loads using 3 or 4 wire system  
 ↳ Balanced → Sources have equal Freq. but different phase  
 ↳ Unbalanced

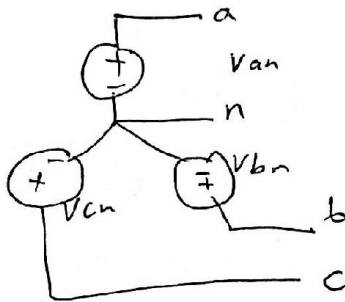
\* 3 phase used in All world to generate, Transmit, Distribute Power.

\* Balanced 3  $\phi$   $\equiv$  3 single phase voltage sources connected either as Y or  $\Delta$  to 3 loads connected as Y or  $\Delta$

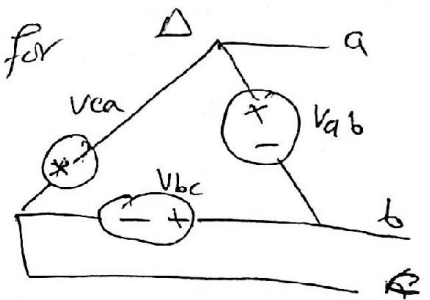


- Y-Y
- Y- $\Delta$
- $\Delta$ -Y
- $\Delta$ - $\Delta$

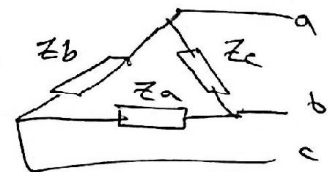
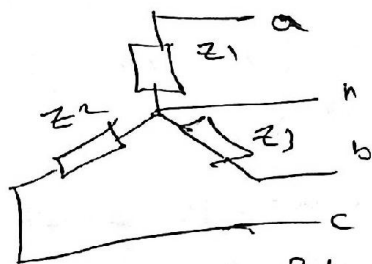
\* ex for Y generators/source



\* ex for



\* ex for Y load



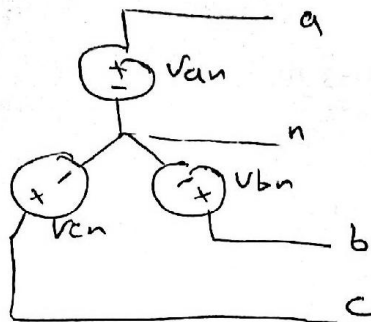
Imp,  $Z_c$   $\sim$   $b$   $a$   $\sim$   
 $Z_b$   $\sim$   $a$   $c$   $\sim$

Notes

$Z_1 = Z_2 = Z_3 = Z_Y$  &  $Z_a = Z_b = Z_c = Z_\Delta$   
 Balanced load      Balanced load

for Balanced load  $Z_Y = \frac{Z_\Delta}{3}$

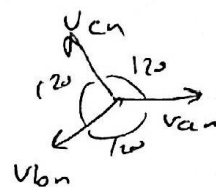
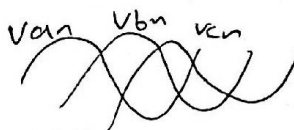
Phase sequence



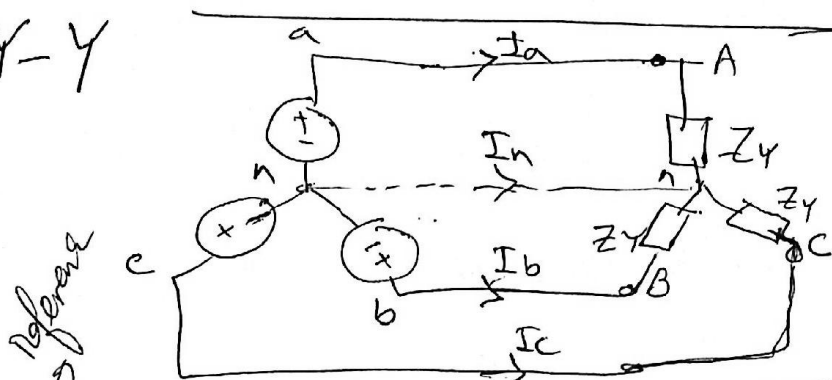
$$V_{an}(t) = \sqrt{2} U_p \cos \omega t = V_p L^0$$

$$V_{bn} = \sqrt{2} U_p \cos(\omega t - 120^\circ) = V_p L^{-120}$$

$$V_{cn} = \sqrt{2} U_p \cos(\omega t + 120^\circ) = V_p L^{120}$$



1 Y-Y



a)  $V_{an} = V_p L^0$   
 $V_{bn} = V_p L^{-120}$   
 $V_{cn} = V_p L^{120}$

phase voltages

b)  $I_a = \frac{V_p L^0}{Z_Y}$   
 $I_b = \frac{V_p L^{-120}}{Z_Y} = I_a L^{-120}$   
 $I_c = \frac{V_p L^{120}}{Z_Y} = I_a L^{120}$

c)  $V_{ab} = V_a - V_b = (V_a - V_n) - (V_b - V_n) = V_{an} - V_{bn}$   
 $= V_p L^0 - V_p L^{-120} = V_p \cos \omega t - V_p \cos(\omega t - 120^\circ)$   
 $= V_p [\cos \omega t - (\cos \omega t \cos 120^\circ + \sin \omega t \sin 120^\circ)]$   
 $= V_p [\cos \omega t + \frac{1}{2} \cos \omega t + \frac{\sqrt{3}}{2} \sin \omega t]$   
 $= V_p [\frac{3}{2} \cos \omega t + \frac{\sqrt{3}}{2} \sin \omega t] = \sqrt{3} V_p [\frac{\sqrt{3}}{2} \cos \omega t + \frac{1}{2} \sin \omega t]$   
 $= V_p \cos(\omega t + \phi) \rightarrow \phi = \tan^{-1}(\frac{\sqrt{3}/2}{\sqrt{3}/2}) = 30^\circ$   
 $= \sqrt{3} V_p \cos(\omega t + 30^\circ) = \sqrt{3} V_p L^{30^\circ}$

$V_{ab} = \sqrt{3} V_p L^{30^\circ}$   
 $V_{bc} = \sqrt{3} V_p L^{-90}$   
 $V_{ca} = \sqrt{3} V_p L^{150}$

phase sequence:  $V_{an} = V_p L^0$ ,  $V_{bn} = V_p L^{-120}$ ,  $V_{cn} = V_p L^{120}$

$V_{LN} = V_L / \sqrt{3} \Rightarrow I_L = \frac{V_{LN}}{Z}$

$V_{BA} = V_B - V_A = V_p L^{120} - V_p L^0$   
 $V_{an} = V_p L^0$   
 $V_{cn} = V_p L^{120}$

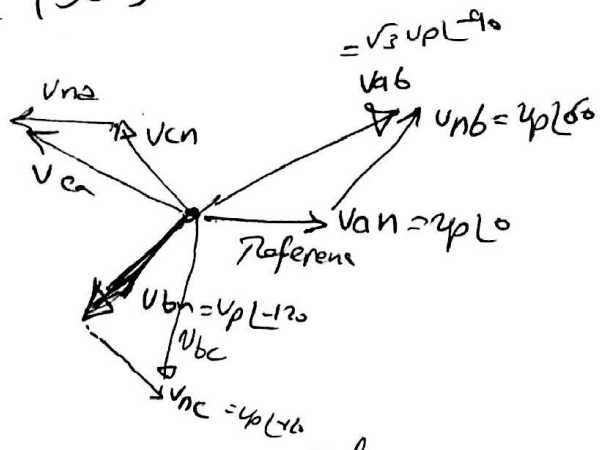
(3)

Phasors

$$V_{ab} = V_{an} + V_{nb} = \sqrt{3} V_p \angle 30^\circ$$

$$V_{bc} = V_{bn} + V_{nc} = \sqrt{3} V_p \angle -90^\circ$$

$$V_{ca} = V_{cn} + V_{na} = \sqrt{3} V_p \angle 150^\circ$$

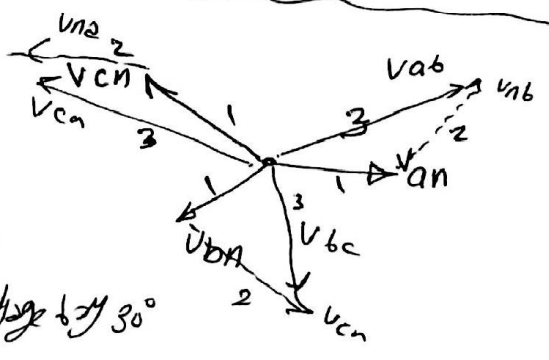


Reference  $V_{ab} \angle 30^\circ = V_{nb} \angle 120^\circ \rightarrow V_{an} \angle 0^\circ$   
 $- V_p \angle 120^\circ = V_{cn} = V_{nc} \angle 120^\circ \leftarrow V_p \angle -120^\circ = V_{bn}$   
 $- V_p \angle 0^\circ = -V_{an} = V_{na} \angle 0^\circ \leftarrow V_p \angle 120^\circ = V_{ca}$

Phase sequence:  $V_{bn} \angle V_{cn} \angle V_{an}$  and  $V_{ca} \angle V_{bc} \angle V_{ab}$

Note: Load voltage  $= V_L = \sqrt{3} V_p$

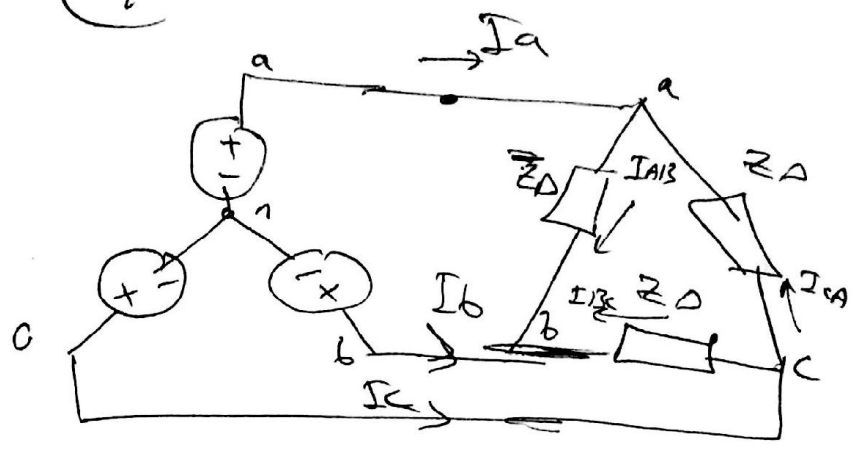
$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}|$   
 and  $V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$   
 Line voltage lead phase voltage by  $30^\circ$



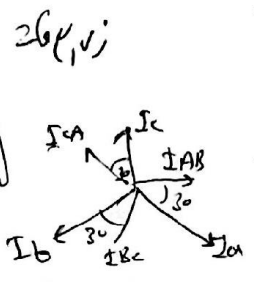
4

2)  $3\phi \rightarrow Y-\Delta$

$\times V_{an} = V_p \angle 0^\circ$   
 $\times V_{bn} = V_p \angle -120^\circ$   
 $\times V_{cn} = V_p \angle 120^\circ$



$\times V_{ab} = \sqrt{3} V_p \angle 30^\circ = V_{AB}$   
 $\times V_{bc} = \sqrt{3} V_p \angle -90^\circ = V_{BC}$   
 $\times V_{ca} = \sqrt{3} V_p \angle 150^\circ = V_{CA}$

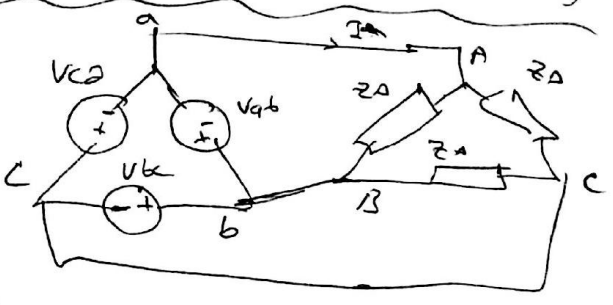


$I_{AB} = \frac{V_{AB}}{Z_{\Delta}}$   
 $I_{BC} = \frac{V_{BC}}{Z_{\Delta}}$   
 $I_{CA} = \frac{V_{CA}}{Z_{\Delta}}$

$I_a = I_{AB} - I_{CA} = \sqrt{3} I_{AB} \angle -30^\circ$   
 $I_b = I_{BC} - I_{AB} = \sqrt{3} I_{BC} \angle -30^\circ$   
 $I_c = \sqrt{3} I_{CA} \angle -30^\circ$

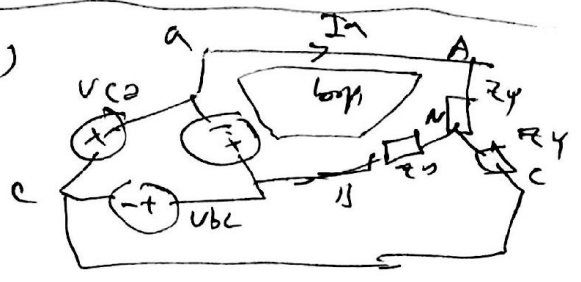
3)  $\Delta-\Delta$

$V_{ab} = V_p \angle 0^\circ = V_{AB} \rightarrow I_{AB} = \frac{V_{AB}}{Z_{\Delta}}$   
 $V_{bc} = V_p \angle -120^\circ = V_{BC} \rightarrow I_{BC} = \frac{V_{BC}}{Z_{\Delta}}$   
 $V_{ca} = V_p \angle 120^\circ = V_{CA} \rightarrow I_{CA} = \frac{V_{CA}}{Z_{\Delta}}$



$I_a = I_{AB} - I_{CA} = \sqrt{3} I_{AB} \angle -30^\circ$   
 $I_b = \sqrt{3} I_{BC} \angle -30^\circ$   
 $I_c = \sqrt{3} I_{CA} \angle -30^\circ$

4)  $\Delta-Y$  (Y-load - Δ-load)



$V_{ab} = V_p \angle 0^\circ$   
 $V_{bc} = V_p \angle -120^\circ$   
 $V_{ca} = V_p \angle 120^\circ$

Loop 1:  $-V_{ab} + Z_y I_a - Z_y I_b = 0$   
 $I_a - I_b = V_{ab} / Z_y$

For Balance  $I_b = I_a \angle -120^\circ$  so  $I_a - I_b = I_a \sqrt{3} \angle 30^\circ = \frac{V_p \sqrt{3}}{Z_y} \angle -30^\circ$

~~For Balance  $I_c = I_b \angle 120^\circ$~~

$P_T = \sqrt{3} V_L I_L \cos \theta$   
 $S_T = \sqrt{3} V_L I_L$   
 $Q_T = \sqrt{3} V_L I_L \sin \theta$

$\therefore I_a = \frac{V_p \sqrt{3}}{Z_y} \angle -30^\circ$   
 $Z_y \angle -120^\circ = I_b \quad \omega t = 2\pi$   
 $I_a \angle +120^\circ = I_c$

5  
Unbalanced

1) 3φ-3wire

$$I_{AB} = V_{AB} / Z_{AB}$$

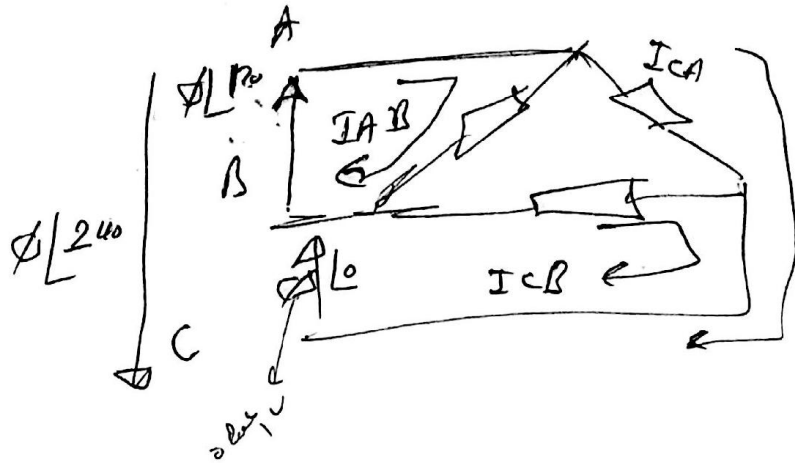
$$I_{BC} = V_{BC} / Z_{BC}$$

$$I_{CA} = V_{CA} / Z_{CA}$$

$$I_A = I_{AB} + I_{AC}$$

$$I_B = I_{BA} + I_{BC}$$

$$I_C = I_{CA} + I_{CB}$$



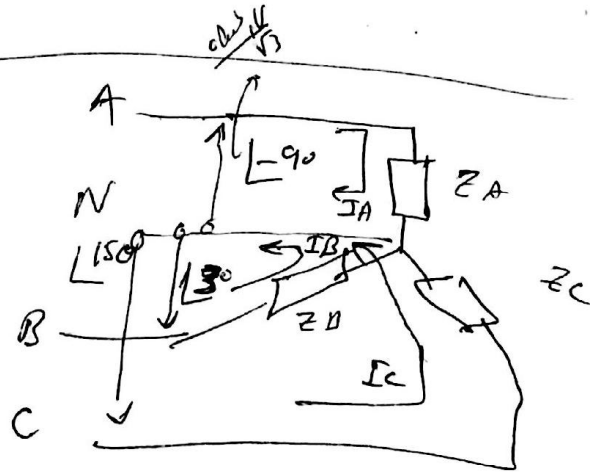
2) 4-wire - Y load

$$I_A = V_{AN} / Z_A$$

$$I_B = V_{BN} / Z_B$$

$$I_C = V_{CN} / Z_C$$

$$I_N = -(I_A + I_B + I_C)$$



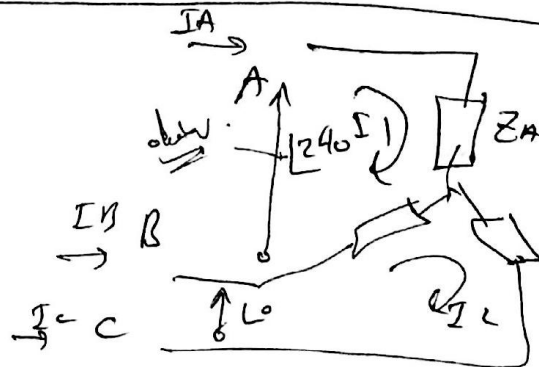
3) 3 wire → Y load

KVL

$$V_{AB} = I_1 Z_A + I_2 Z_B$$

$$V_{BC} = (I_2 - I_1) Z_B + I_2 Z_C$$

← KVL loop equations



$$I_A = I_1$$

$$I_B = I_2 - I_1$$

$$I_C = -I_2$$

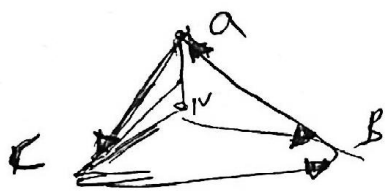
→ KVL loops

→ Power input, output  
→ comparison

6 Use 300

Phase in  $\Delta$  Ref. Reference Seqs.  $\Delta$   $\rightarrow$   $\Delta$   $\rightarrow$   $\Delta$   
 Use  $\Delta$  Ref for balanced load

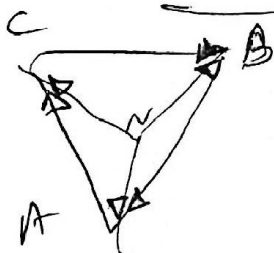
ABC seq



$V_{AB} = V_L \angle 120^\circ$   
 $V_{BC} = V_L \angle 0^\circ$   
 $V_{CA} = V_L \angle 240^\circ$

$V_{AN} = \frac{V_L}{\sqrt{3}} \angle 90^\circ$   
 $V_{BN} = \frac{V_L}{\sqrt{3}} \angle -30^\circ$   
 $V_{CN} = \frac{V_L}{\sqrt{3}} \angle -150^\circ$

CBA seq



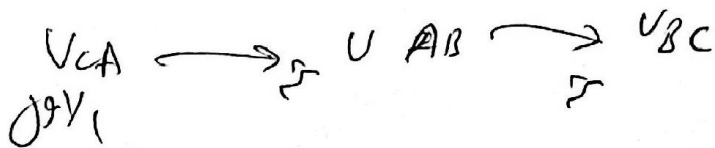
$V_{AB} = V_L \angle 240^\circ$   
 $V_{BC} = V_L \angle 120^\circ$   
 $V_{CA} = V_L \angle 0^\circ$

$V_{AN} = \frac{V_L}{\sqrt{3}} \angle -90^\circ$   
 $V_{BN} = \frac{V_L}{\sqrt{3}} \angle 30^\circ$   
 $V_{CN} = \frac{V_L}{\sqrt{3}} \angle 150^\circ$

300

4 wire

	AB Ref	BC Ref	CA Ref
$V_{AB} =$	$V_L \angle 0^\circ$	$V_L \angle 120^\circ$	$V_L \angle -120^\circ$ or $V_L \angle 240^\circ$
$V_{BC} =$	$V_L \angle -120^\circ$	$V_L \angle 0^\circ$	$V_{BC} \angle -240^\circ$ or $V_L \angle 120^\circ$
$V_{CA} =$	$V_L \angle +120^\circ$	$V_L \angle 240^\circ$	$V_L \angle 0^\circ$ or $V_L \angle 360^\circ$



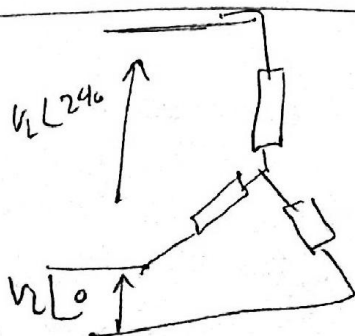
For Balanced

$P = \sqrt{3} V_L I_L \cos \theta_L$

$\rightarrow$  angle bet

- $V_{AB}, I_{AB}$
- $V_{BC}, I_{BC}$
- $I_{CA}, V_{CA}$

For 3 wire  $\Delta$



8

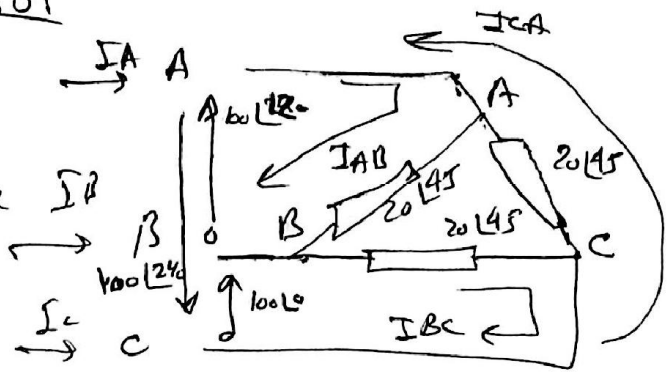
10  $\hat{a} \hat{a} \hat{a}$  A

1) 3  $\phi$  3 wire \* 100V \* ABC sys supplies a balanced  $\Delta$  load with impedance  $20 \angle 45^\circ$  find  $I_{Line}$  draw phasors

801

~~$I_{AB} = \frac{V_{AB}}{Z}$~~   
 Let VBC Reference  $100 \angle 0^\circ$   
 Exp:  $V_{AB} = 100 \angle 120^\circ$   
 $V_{CA} = 100 \angle 240^\circ$

phase voltage = load voltage



Phase current

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{100 \angle 120^\circ}{20 \angle 45^\circ} = 5 \angle 75^\circ$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{100 \angle 0^\circ}{20 \angle 45^\circ} = 5 \angle -45^\circ$$

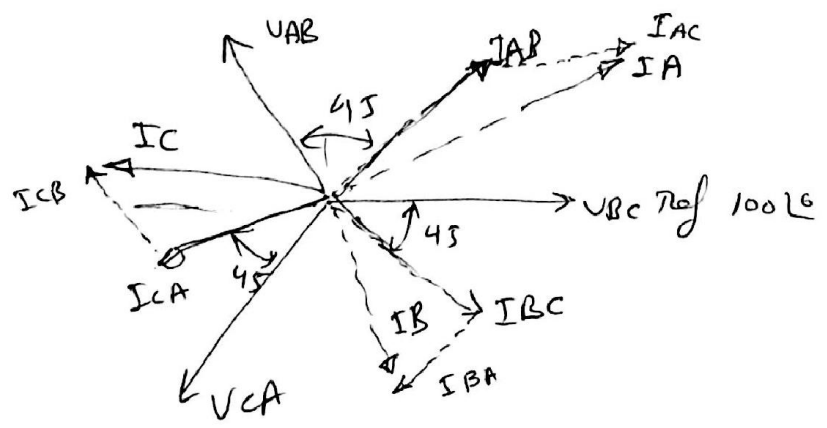
$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{100 \angle 240^\circ}{20 \angle 45^\circ} = 5 \angle 195^\circ$$

load current

$$I_A = I_{AB} - I_{CA} = I_{AB} + I_{AC} = 5 \angle 75^\circ - 5 \angle 195^\circ = 8.66 \angle 45^\circ$$

$$I_B = I_{BC} - I_{AB} = I_{BC} + I_{BA} = 5 \angle -45^\circ - 5 \angle 75^\circ = 8.66 \angle -75^\circ$$

$$I_C = I_{CA} - I_{BC} = 5 \angle 195^\circ - 5 \angle -45^\circ = 8.66 \angle 165^\circ$$



$$P = \sqrt{3} V_L I_L \cos \theta$$

$\swarrow$  100     $\downarrow$  8.66     $\searrow$  45

$I_{AB} \angle V_{AB} = 45^\circ$   
 $I_{BC} \angle V_{BC}$   
 $I_{CA} \angle V_{CA}$



(8)

2] 8 identical impedances of  $5L-30$ , connected in Y to  $3\phi$  of 3 wire 150V, CBA sy find IL, Phases

for CBA sy

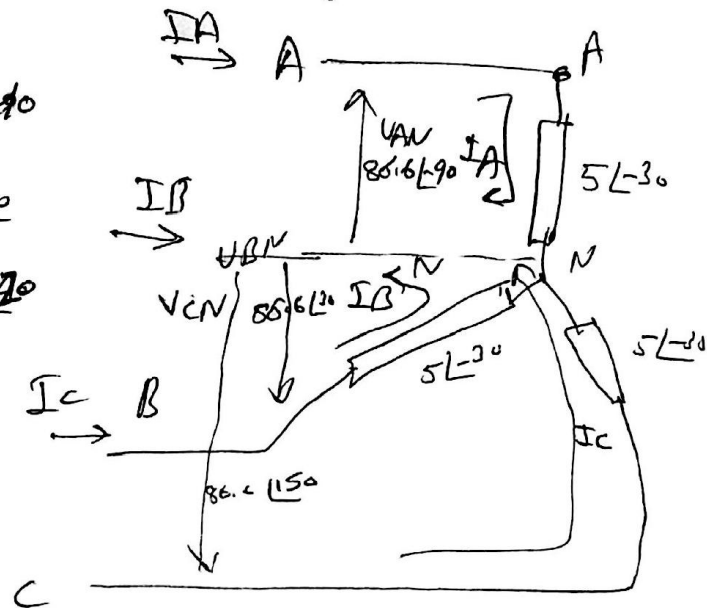
reference

$$V_{AN} = V_L / \sqrt{3} \angle 90^\circ, \quad V_{AB} = V_L \angle 120^\circ$$

$$V_{BN} = V_L / \sqrt{3} \angle +30^\circ, \quad V_{BC} = V_L \angle 0^\circ$$

$$V_{CN} = V_L / \sqrt{3} \angle +150^\circ, \quad V_{CA} = V_L \angle 270^\circ$$

$$V_{LN} = V_L / \sqrt{3} = 150 / \sqrt{3} = 86.6$$

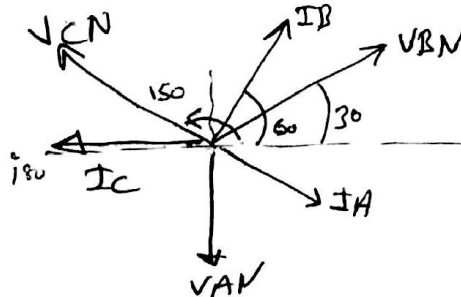


$$I_A = V_{AN} / 5L-30$$

$$I_A = \frac{V_L / \sqrt{3} \angle -90^\circ}{5L-30} = \frac{150 / \sqrt{3}}{5} \angle -9 - (-30) = 17.3 \angle -60$$

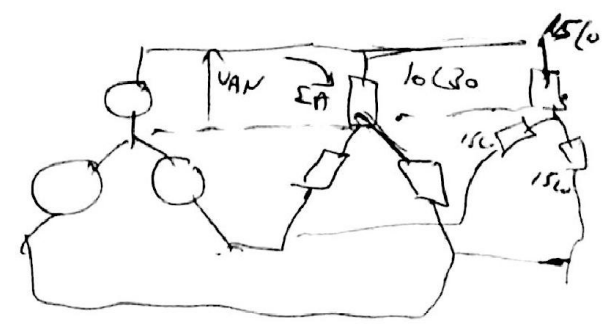
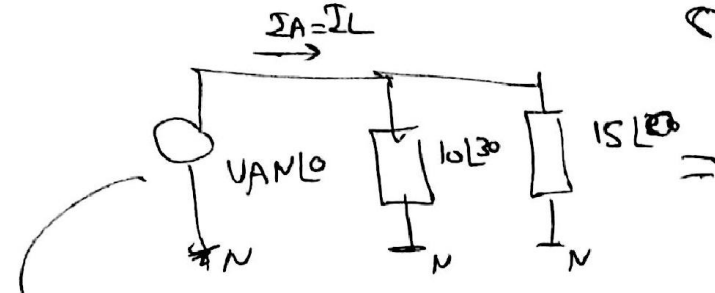
$$I_B = \frac{V_{BN}}{5L-30} = \frac{V_L / \sqrt{3} \angle 30^\circ}{5L-30} = 17.32 \angle 60$$

$$I_C = \frac{V_{CN}}{Z} = \frac{V_L / \sqrt{3} \angle 150^\circ}{5L-30} = 17.32 \angle 180$$



9

3) 3 identical impedances of  $10 \angle 30^\circ$  in Y connection & 3 impedances of  $15 \angle 0^\circ$  also in Y are both in same 3 phase, 250 V rms of total power



$$|V_{AN}| = \frac{|V_L|}{\sqrt{3}} = \frac{250}{\sqrt{3}} = 144.5$$

$$P = \sqrt{3} V_L I_L \cos \theta$$

$$I_L = I_A = I_{10 \angle 30^\circ} + I_{15 \angle 0^\circ} = \frac{V_{AN}}{10 \angle 30^\circ} + \frac{V_{AN}}{15 \angle 0^\circ}$$

for ABC seq

فصل اول  
تفاضل حاد  
P.T. 13  
مركب  
دائرة

$$I_L = \frac{144.5 \angle 0^\circ}{10 \angle 30^\circ} + \frac{144.5 \angle 0^\circ}{15 \angle 0^\circ} = 23.2 \angle -18.1^\circ$$

$V_{AN} \angle 0^\circ$   
 $V_{BN} \angle -120^\circ$   
 $V_{CN} \angle 120^\circ$

$$P = \sqrt{3} V_L I_L \cos \theta = \sqrt{3} (250) (23.2) \cos(-18.1^\circ) = 9530 \text{ w}$$

$\sum V_L = \text{الخط}$

Another Ans

$$V_{AN} = 250 \angle 9^\circ$$

$$V_{BN} = 250 \angle -30^\circ$$

$$V_{CN} = 250 \angle -150^\circ$$

مركب و خط

$$I_L = \frac{250 \angle 9^\circ}{10 \angle 30^\circ} + \frac{250 \angle 9^\circ}{15 \angle 0^\circ} = 23.27 \angle 71.9^\circ$$

$$\theta_{\text{impedance}} = |\theta_i - \theta_v| = 90 - 71.9 = 18.1$$

$$P_L = \sqrt{3} V_L I_L \cos \theta = \sqrt{3} \times 250 \times 23.27 \times \cos 18.1^\circ = 9530 \text{ w}$$

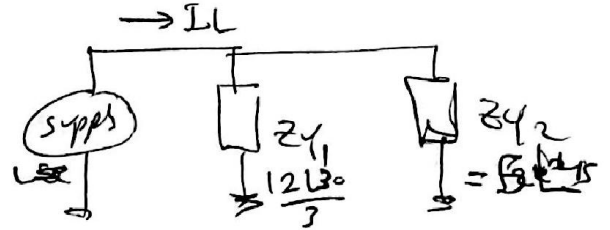
4) 3 identical impedances of  $12 \angle 30^\circ$  in  $\Delta$  connection, and 3 identical  $\sim$  of  $5 \angle 45^\circ$  in  $Y$ , connected across 3 wire, 3 $\phi$  (208V) ABC sys  $\rightarrow$  find  $I_L, P_{total}$

Sol

مقاومة متساوية، في  $\Delta$  و  $Y$ ،  $Z_{\Delta} = Z_Y$  ←  $Y \rightarrow \Delta$  لسهولة

1)  $Z_Y = \frac{12 \angle 30^\circ}{3} = 4 \angle 30^\circ$

2)  $V_{supps} = \frac{V_{line}}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120V$



3)  $I_L = \frac{120 \angle 0^\circ}{4 \angle 30^\circ} + \frac{120 \angle 0^\circ}{5 \angle 45^\circ} = 53.6 \angle -36.6^\circ$

in ABC

$\therefore V_{AN} = \frac{V_L}{\sqrt{3}} \angle 90^\circ$   $\therefore I_A = \frac{V_{AN}}{Z_{eq}} = \frac{120 \angle 90^\circ}{2.24 \angle 36^\circ} = 53.6 \angle 53.4^\circ$

$I_B = \frac{V_{BN}}{Z_{eq}} = \frac{120 \angle -3^\circ}{2.24 \angle 36^\circ} = 53.6 \angle -66.6^\circ$

$I_C = \frac{V_{CN}}{Z_{eq}} = \frac{120 \angle -150^\circ}{2.24 \angle 36^\circ} = 53.6 \angle -186.6^\circ$

$V_{AN}, V_{BN}, V_{CN} \rightarrow$  for ABC sys

في  $ABC$  النظام 4 (3)  $\angle$  في  $Y$   $\rightarrow$   $Z_{eq}$   $\rightarrow$   $Z_{eq} = 2.24 \angle 36^\circ$

$P_t = \sqrt{3} V_L I_L \cos \theta = \sqrt{3} (208)(53.6) \cos (36.6^\circ)$

$\rightarrow$   $V_L \angle I_L = \theta$

5) 3φ, 3 wire, 240V, CBA sys, supply Δ connected load in which  $Z_{AB} = 25 \angle 90^\circ$ ,  $Z_{BC} = 15 \angle 30^\circ$ ,  $Z_{CA} = 20 \angle 0^\circ$  find line current & total power

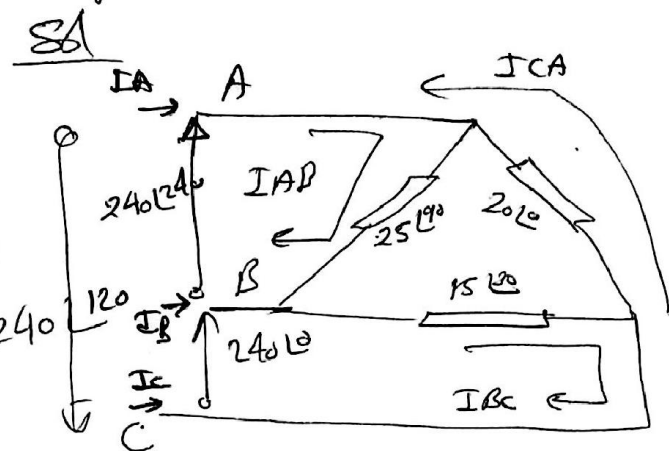
Sol

Phase current

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{240 \angle 240^\circ}{25 \angle 90^\circ} = 9.6 \angle 150^\circ$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{240 \angle 0^\circ}{15 \angle 30^\circ} = 16 \angle -30^\circ$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{240 \angle 120^\circ}{20 \angle 0^\circ} = 12 \angle 120^\circ$$



For CBA

$$I_A = I_{AB} - I_{CA} = I_{AB} + I_{CA} = 6.6 \angle 247.2^\circ$$

$$I_B = I_{BC} - I_{AB} = I_{BC} + I_{BA} = 25.6 \angle -30^\circ$$

$$I_C = I_{CA} - I_{BC} = 27.1 \angle 137.2^\circ$$

→ to calc. Power in load

$$P_T = P_{load1} + P_{load2} + P_{load3}$$

$$= I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3$$

$$= I_{AB}^2 Z_{AB} + I_{BC}^2 Z_{BC} + I_{CA}^2 Z_{CA}$$

$$= (9.6)^2 \times 0 + (16)^2 \times 13 + (12)^2 \times 20 = 6210W$$

$$Re Z_1 = Re(25 \angle 90^\circ) = 25 \cos 90^\circ = 0$$

$$Z_2 = 15 \angle 30^\circ = 13 + j7.5$$

$$Z_3 = 20 \angle 0^\circ = 20$$

6) 3 $\phi$ , Four wire, 208V, ABC, Y connected load  
 $Z_A = 10 \angle 0^\circ$ ,  $Z_B = 15 \angle 30^\circ$ ,  $Z_C = 10 \angle -30^\circ$ , find  $I_N$   
 $I_N$ ,  $P_t$

Sol

Take  $V_{BC}$  Ref  $\rightarrow$  For ABC sys  $\rightarrow$

$$I_A = V_{AN} / 10 \angle 0^\circ = 120 \angle 90^\circ / 10 \angle 0^\circ = 12 \angle 90^\circ$$

$$I_B = V_{BN} / 15 \angle 30^\circ = 120 \angle -30^\circ / 15 \angle 30^\circ = 8 \angle -60^\circ$$

$$I_C = V_{CN} / 10 \angle -30^\circ = 120 \angle 150^\circ / 10 \angle -30^\circ = 12 \angle 120^\circ$$

$$I_N = -(I_A + I_B + I_C) = 5.64 \angle 69.4^\circ$$

$$V_{\phi} = \frac{208}{\sqrt{3}} = 120$$

$$P_t = P_A + P_B + P_C = I_A^2 Z_A + I_B^2 Z_B + I_C^2 Z_C$$

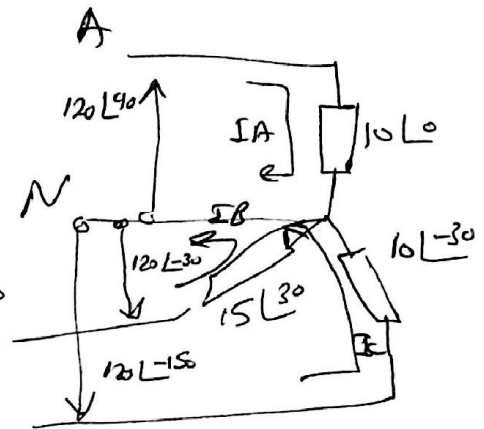
5)  $I_C$  is  $\angle 120^\circ$  relative to  $I_A$  and  $I_B$  is  $\angle -60^\circ$  relative to  $I_A$

$$P_A = I_A^2 Z_A = (12)^2 \times \text{Re} Z_A = (144) \times (10) = 1440$$

$$P_B = I_B^2 Z_B = (8)^2 \times \text{Re} Z_B = (64) \left( \frac{15 \cos 30^\circ}{15} \right) = 832$$

$$P_C = I_C^2 Z_C = (12)^2 \times \text{Re} Z_C = (144) \left( \frac{10 \cos -30^\circ}{10} \right) = 1247$$

$$\therefore P_t = 1440 + 832 + 1247 = 3519 \text{ W}$$



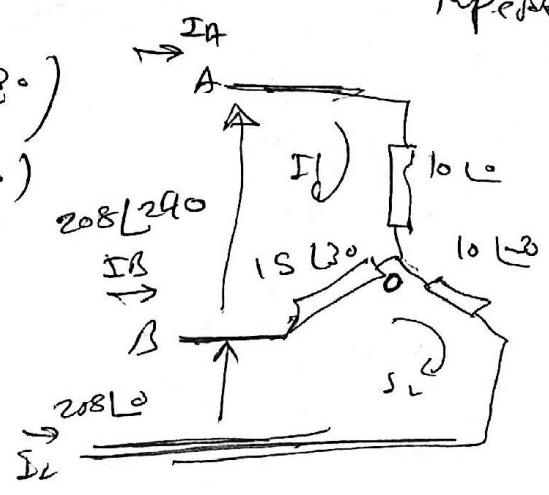
[7] The load impedance of Prob (6) connected to 3 $\phi$ , 3 wire 208 V ABC find the line and end voltage across load impedances

Use Mesh

$$208 \angle 240^\circ = I_1(10 \angle 0^\circ + 15 \angle 30^\circ) - I_2(15 \angle 30^\circ)$$

$$208 \angle 0^\circ = I_2(10 \angle -30^\circ + 15 \angle 30^\circ) - I_1(15 \angle 30^\circ)$$

Phase  
 $I_1 = 14.15 \angle 86.1^\circ$   
 $I_2 = 10.15 \angle 52.7^\circ$



Line current  
 $I_A = I_1 = 14.15 \angle 86.1^\circ$   
 $I_B = I_2 - I_1 = 8 \angle -49.5^\circ$   
 $I_C = -I_2 = 10.15 \angle -127.3^\circ$

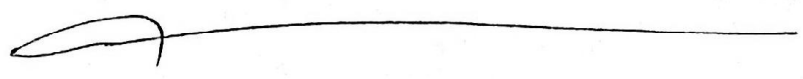
$$-I_2 = -10.15 \angle 52.7^\circ = -10.15 \angle 52.7^\circ + j10.15 \sin 52.7^\circ$$

Phase voltage

$$V_{A0} = V_{Z1} = I_1 \times 10 \angle 0^\circ = (14.15 \angle 86.1^\circ)(10 \angle 0^\circ) = 141.5 \angle 86.1^\circ$$

$$V_{Z2} = (I_2 - I_1)(15 \angle 30^\circ) = (8 \angle -49.5^\circ)(15 \angle 30^\circ) = 120 \angle -19.5^\circ$$

$$V_{Z3} = I_2 \times 10 \angle -30^\circ = (10.15 \angle -127.3^\circ)(10 \angle -30^\circ) = 101.5 \angle -157.3^\circ$$



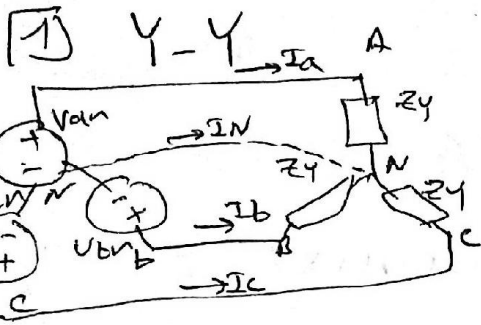
# Three phase $\omega \cos \omega t$ analysis 1 - Balanced sys

connection

Phase Voltage  
current

Line voltage  
current

$$I_n + I_a + I_b + I_c = 0$$



$$V_{an} = V_p \angle 0$$

$$V_{bn} = V_p \angle -120$$

$$V_{cn} = V_p \angle +120$$

I is the same line

$I_{AB}, I_{BC}, I_{CA}$

$$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}| = \sqrt{3} V_p$$

$$V_{ab} = \sqrt{3} V_p \angle 30$$

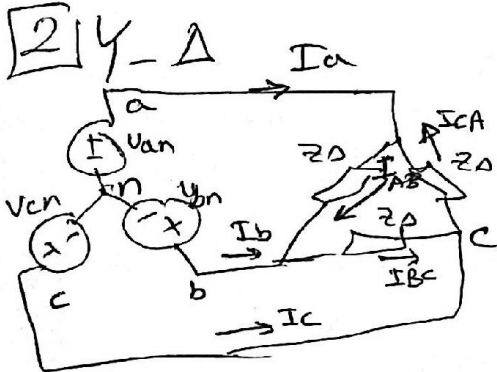
$$V_{bc} = V_{ab} \angle -120$$

$$V_{ca} = V_{ab} \angle +120$$

$$I_a = V_{an} / Z_Y$$

$$I_b = I_a \angle -120 = \frac{V_{bn}}{Z_Y}$$

$$I_c = I_a \angle 120 = \frac{V_{cn}}{Z_Y}$$



$$V_{an} = V_p \angle 0$$

$$V_{bn} = V_p \angle -120$$

$$V_{cn} = V_p \angle +120$$

$$I_{AB} = V_{ab} / Z_{\Delta}$$

$$I_{BC} = V_{bc} / Z_{\Delta}$$

$$I_{CA} = V_{ca} / Z_{\Delta}$$

$$V_{ab} = V_{AB} = \sqrt{3} V_p \angle 30$$

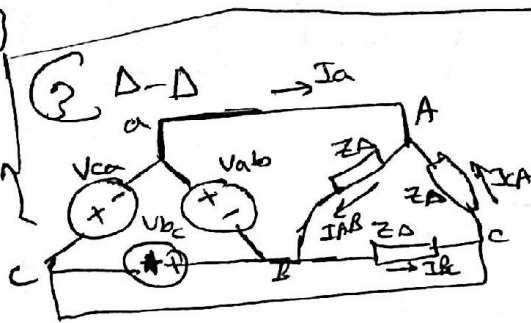
$$V_{bc} = V_{ab} \angle -120$$

$$V_{ca} = V_{ab} \angle 120$$

$$I_a = I_{AB} \sqrt{3} \angle -30 = \frac{I_{AB}}{I_{CA}}$$

$$I_b = I_a \angle -120 = \frac{I_{BC}}{I_{AB}}$$

$$I_c = I_a \angle 120$$



$$V_{ab} = V_p \angle 0$$

$$V_{bc} = V_p \angle -120$$

$$V_{ca} = V_p \angle 120$$

$$I_{AB} = V_{ab} / Z_{\Delta}$$

$$I_{BC} = V_{bc} / Z_{\Delta}$$

$$I_{CA} = V_{ca} / Z_{\Delta}$$

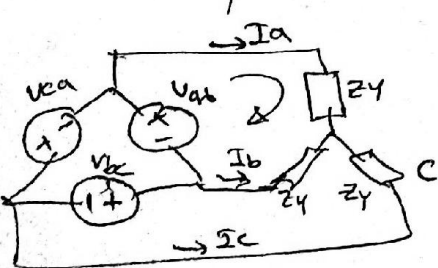
same phase  $\angle 30$

$$I_a = I_{AB} \sqrt{3} \angle -30 = I_{AB} - I_{CA}$$

$$I_b = I_a \angle -120 = I_{BC} - I_{AB}$$

$$I_c = I_a \angle 120$$

4  
Δ-Y



$$V_{ab} = V_p \angle 0$$

$$V_{bc} = V_p \angle -120$$

$$V_{ca} = V_p \angle 120$$

same direction

same phase voltage

$$I_a = \frac{V_p}{\sqrt{3}} \frac{\angle -30}{Z_Y}$$

$$I_b = I_a \angle -120$$

$$I_c = I_a \angle 120$$

for Δ or Y  
Balanced

$$P_T = \sqrt{3} V_L I_L \cos \theta$$

$$P_T = \sqrt{3} V_L I_L \cos \theta$$

$$P_T = \sqrt{3} V_L I_L \cos \theta$$

where in Δ  $I_L = \sqrt{3} I_p$  & in Y  $V_L = \sqrt{3} V_p$

① 3φ-3wire Δ ABC or CBA sys  
 $V_{line} = 240$

$I_{AB} = V_{AB} / Z_{AB}$

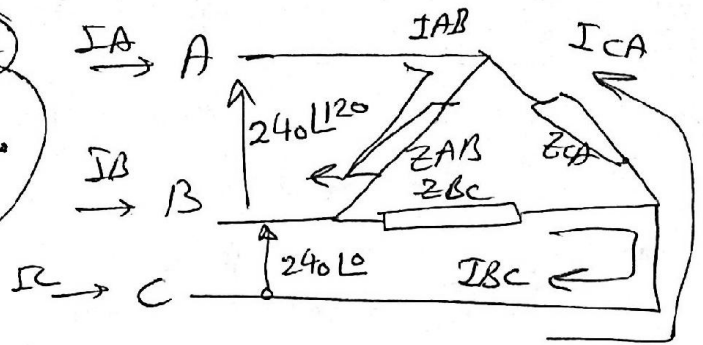
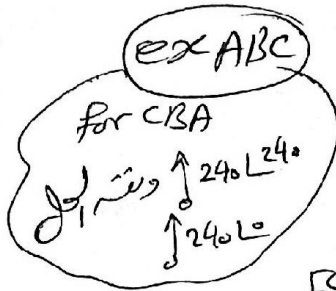
$I_{BC} = V_{BC} / Z_{BC}$

$I_{CA} = \frac{V_{CA}}{Z_{CA}}$

$I_A = I_{AB} - I_{CA}$

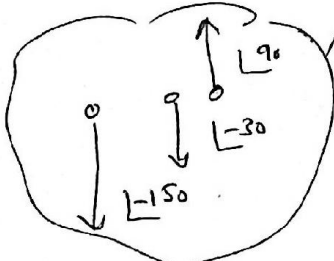
$I_B = I_{BC} - I_{AB}$

$I_C = I_{CA} - I_{BC}$

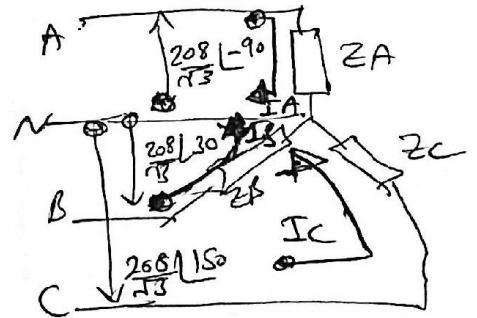


② 4 wire - Y ABC or CBA ex (CBA)  $V_L = 208V$

رابطه بین ولتاژ خطی و ولتاژ فاز در سیستم 4 سیم و 3 فاز  
 $V_L = \sqrt{3} V_{ph}$



ABC فازها  
 $I_A = V_{AN} / Z_A$   
 $I_B = V_{BN} / Z_B$   
 $I_C = V_{CN} / Z_C$



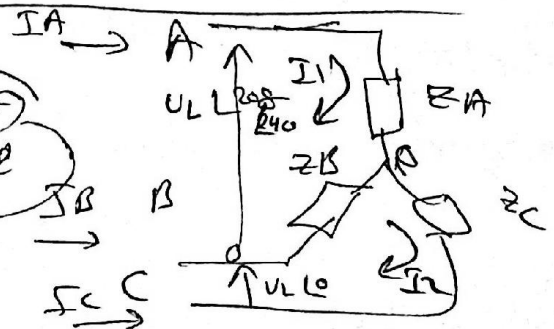
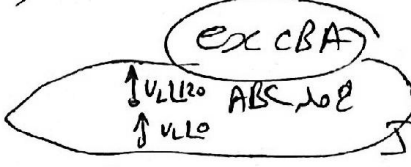
نول به مرکز بار وصل می شود

$I_A = V_{AN} / Z_A$  ,  $I_B = V_{BN} / Z_B$  ,  $I_C = V_{CN} / Z_C$

$I_N + I_A + I_B + I_C = 0$

③ 3φ, 3wire Y (ABC) or (CBA)

meshloop



$208 \angle 240^\circ = I_1(Z_A + Z_B) - I_2 Z_B$

$208 \angle 0^\circ = I_2(Z_B + Z_C) - I_1 Z_B$

حل سیستم معادلات

$I_A = I_1$

$I_B = I_2 - I_1$

$I_C = -I_2$

$V_{A0} = V_{Z1} = I_1 Z_A$

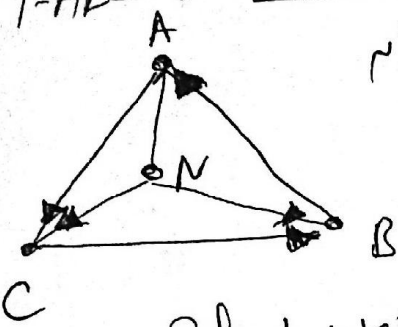
$V_{B0} = V_{Z2} = (I_2 - I_1) Z_B$

$V_{C0} = (I_2) Z_C$



جواب سؤال 34 في الفصل 14

1- ABC sys



نظام 3 wire ABC sys

Ref  $V_{CB}$   
 Ref  $V_{CA}$   
 وندرجها في الجدول

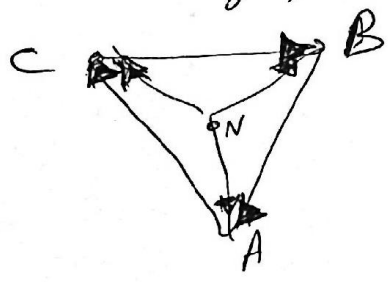
$$\begin{aligned} V_{AB} &= V_L \angle 120^\circ \\ V_{BC} &= V_L \angle 0^\circ \\ V_{CA} &= V_L \angle 240^\circ \end{aligned}$$

4 wire ABC sys

$$\begin{aligned} V_{AN} &= \left(\frac{V_L}{\sqrt{3}}\right) \angle 90^\circ \\ V_{BN} &= \left(\frac{V_L}{\sqrt{3}}\right) \angle -30^\circ \\ V_{CN} &= \left(\frac{V_L}{\sqrt{3}}\right) \angle 150^\circ \end{aligned}$$

وإذا كان لدينا 3 wire في نظام 4 wire...

2- CBA sys



3 wire

$$\begin{aligned} V_{AB} &= V_L \angle 240^\circ \\ V_{BC} &= V_L \angle 0^\circ \\ V_{CA} &= V_L \angle 120^\circ \end{aligned}$$

4 wire

$$\begin{aligned} V_{AN} &= \left(\frac{V_L}{\sqrt{3}}\right) \angle -90^\circ \\ V_{BN} &= \left(\frac{V_L}{\sqrt{3}}\right) \angle +30^\circ \\ V_{CN} &= \left(\frac{V_L}{\sqrt{3}}\right) \angle +150^\circ \end{aligned}$$

Balanced وبتطبيق في الجدول

→ unbalanced

(ABC) (CBA) (ABC) (CBA)

Δ load 3 wire / CBA

Y load 4 wire / CBA

CBA/ABC - three wire with load in Δ

CBA/ABC - three wire with Y load

CBA 3 wire

في A-A

Δ

وإذا